# Minimal trap spaces of Boolean models are maximal siphons of their Petri net encoding

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# Boolean modeling

Boolean modelling of **gene regulation** but also of other biological systems has had great successes over the last ∼20 years.

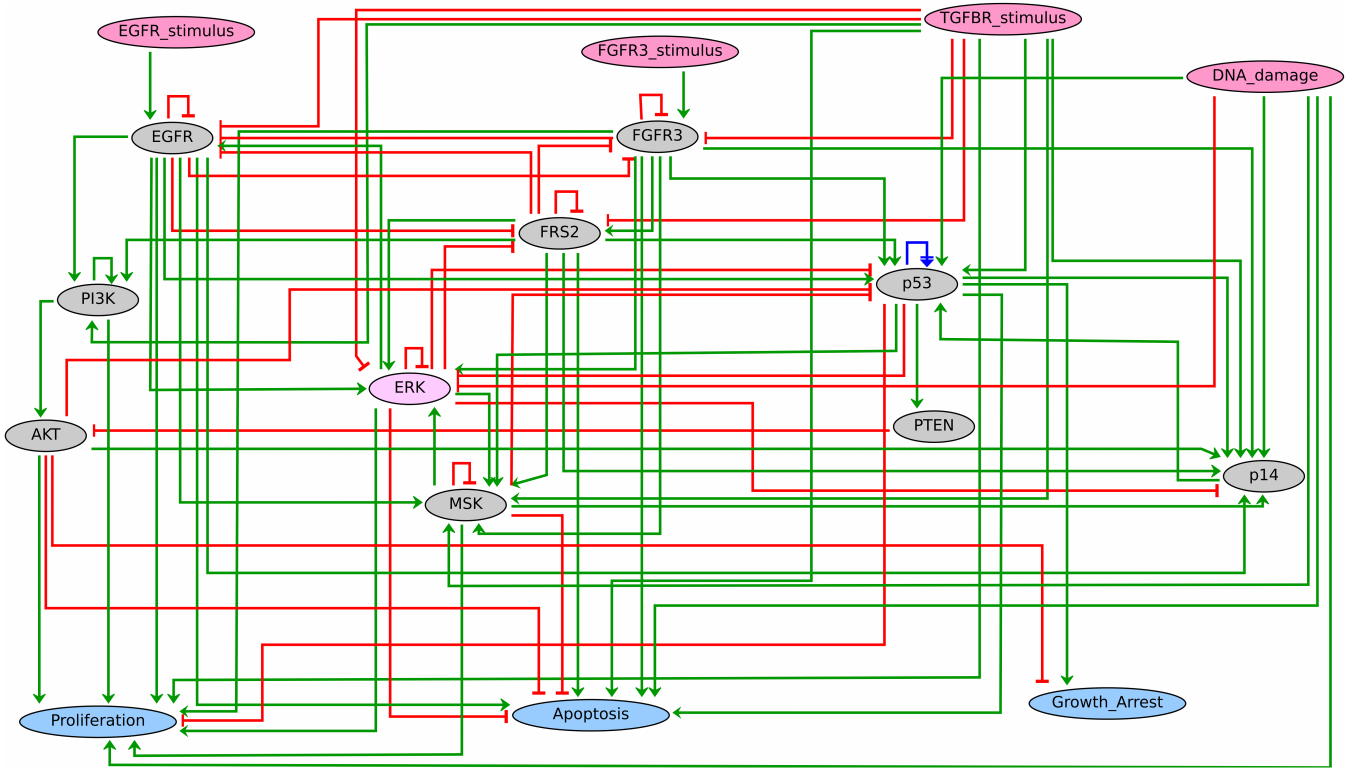



Figure: Boolean model of the MAPK regulatory network, whose involvement in bladder cancer is well established [Grieco et al., 2013].

# Boolean models

## Boolean model

A Boolean model $\mathcal{M}$ is defined as a 2-tuple $(V, F)$, where $V = \{x_1, ..., x_n\}$ $(n \geq 1)$ is a set of nodes and $F = \{f_1, ..., f_n\}$ is a set of Boolean functions. Each node $x_i$ is identified as a Boolean variable, and is associated with a Boolean function $f_i : \mathbb{B}^{|IN(f_i)|} \to \mathbb{B}$, where $IN(f_i)$ is the set of input nodes of $f_i$.

A state $s$ is a mapping $s\colon V \mapsto \mathbb{B}$ that assigns either 0 (inactive) or 1 (active) to each node.

The state space of $\mathcal{M}$ is $\mathbb{B}^n$.

# Dynamics of Boolean models

At each time step $t$, node $x_i$ can update its state by

$$x_i(t+1) = f_i(\mathbf{x}(t)).$$

An update scheme specifies which node will be updated.

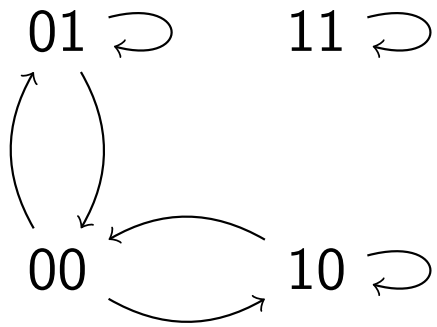
Based on the update scheme, the Boolean model can transit from a state to another state (possibly identical). This is the *state transition* (denoted by $\longrightarrow$).

The dynamics of a Boolean model is captured by a *State Transition Graph* (STG) that is a directed graph whose nodes represent states and whose arcs represent the state transitions.

# Example Boolean model

$$\begin{cases} f_1 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \\ f_2 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \end{cases}$$

Boolean model




State transition graph

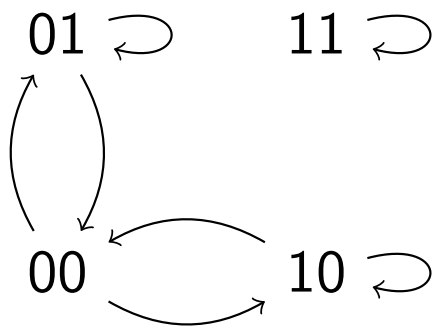
Fully asynchronous update scheme: only one node is nondeterministically selected in order to be updated at each time step.

# Trap sets and attractors

A *trap set* is a non-empty set $S$ of states s.t. $\forall x \longrightarrow y, x \in S \Rightarrow y \in S$.

An *attractor* of a Boolean model is defined as a minimal trap set that does not contain any other trap set as a subset.

$$\begin{cases} f_1 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \\ f_2 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \end{cases}$$




| State set | Trap set? | Attractor? |
|---|---|---|
| $\{11\}$ | yes | yes |
| $\{00, 01\}$ | no | no |
| $\{00, 01, 10\}$ | yes | yes |
| $\{00, 01, 10, 11\}$ | yes | no |

# Application

Besides simulation, the analysis of Boolean models is mostly based on *attractor* computation, since those correspond roughly to observable biological *phenotypes*.

Analysis of attractors could provide new insights into systems biology [Wang et al., 2012] (e.g., the origins of cancers [Montagud et al., 2021], SARS-CoV-2 [Ostaszewski et al., 2021], HIV [Oyeyemi et al., 2014]).

Attractor computation also gives a starting point for many control approaches for biological systems [Fontanals et al., 2020], which play an important role in the development of new drugs [Balbas-Martinez et al., 2018].

# Motivation

Attractor computation of Boolean models is very challenging [Mizera et al., 2019].

The recent use of *trap spaces* as very good approximations of attractors made a real breakthrough in that field allowing to consider medium-sized models that used to be out of reach [Klarner et al., 2015].

However, with the continuing increase in model-size, the state-of-the-art computation of minimal trap spaces based on *prime-implicants* (e.g., PyBoolNet [Klarner et al., 2017]) shows its limits as there can be a huge number of implicants.

The recent method presented in [Chevalier et al., 2019] for computing minimal trap spaces avoids the prime-implicants computation. It is implemented in the tool mpbn [Paulevé et al., 2020] and can handle very large but only *locally-monotonic* Boolean models.

# Contribution

In this work [Trinh et al., 2022], we

- make a connection between trap spaces of Boolean models and siphons of Petri nets, which has not yet been explored before;
- and then propose a novel method to compute minimal trap spaces, and hence attractors, of a Boolean model.

Note that these results are applicable for general Boolean models (i.e., both locally-monotonic and non-locally-monotonic ones).

# Subspaces

A *subspace* $m$ is defined as a mapping $m : V \mapsto \mathbb{B} \cup \star$.

The set of fixed variables of $m$ (denoted by $D_m$) is defined by $D_m := \{v \mid v \in V, m(v) \neq \star\}$.

The set of free variables of $m$ is simply $V \setminus D_m$.

$m$ is equivalent to a set of states $S[m] := \{s \in \mathbb{B}^n \mid \forall x_i \in D_m : s(x_i) = m(x_i)\}$.

For example, $m = 01\star$ means that $D_m = \{x_1, x_2\}$, $m(x_1) = 0$, $m(x_2) = 1$, $m(x_3) = \star$, and $m$ refers to the set of states $\{010, 011\}$.

# Minimal trap spaces

A *trap space* is a set $S$ of states that is a subspace and also a trap set.

A trap space is *minimal* if it does not contain any smaller trap space.

| State set | Trap set? | Subspace? | Trap space? |
|---|---|---|---|
| $\{11\}$ | yes | 11 | yes |
| $\{00, 01\}$ | no | $0\star$ | no |
| $\{00, 01, 10\}$ | yes | no | no |
| $\{00, 01, 10, 11\}$ | yes | $\star\star$ | yes |

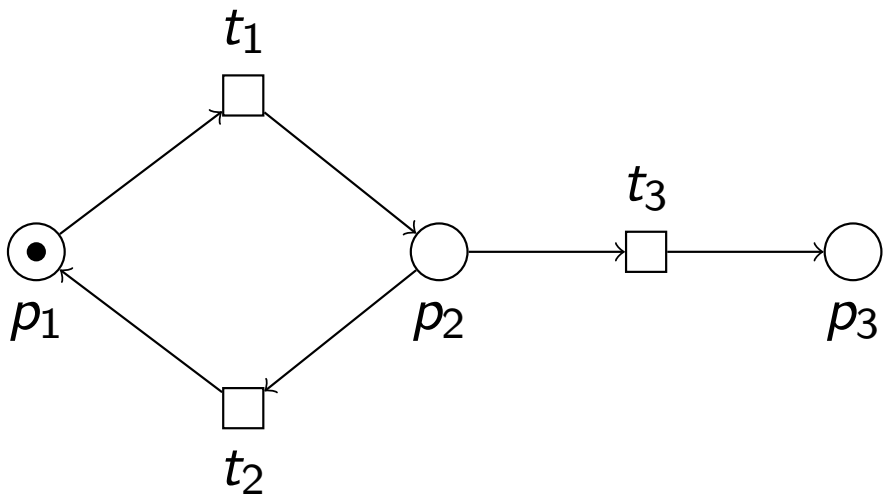
Note that trap spaces of a Boolean model are independent of the update scheme of this model [Klarner et al., 2015].

Each minimal trap space contains at least one attractor, thus minimal trap spaces are good approximations for attractors of a Boolean model.

# Petri nets

Bipartite graph

Places $P$
Transitions $T$
Weighted arcs $W$




A *marking* for a Petri net is a mapping $m : P \mapsto \mathbb{N}$ that assigns a number of tokens to each place. A place $p$ is marked by a marking $m$ if and only if $m(p) > 0$. For example, $p_1$ is marked, $p_2$ and $p_3$ are unmarked.

We shall write $pred(x)$ (resp. $succ(x)$) to represent the set of vertices that have a (non-zero weighted) arc leading to (resp. coming from) $x$. For example, $pred(p_2) = \{t_1\}$, $succ(p_2) = \{t_2, t_3\}$, $pred(t_2) = \{p_2\}$, and $succ(t_2) = \{p_1\}$.

# Siphons

## Siphon

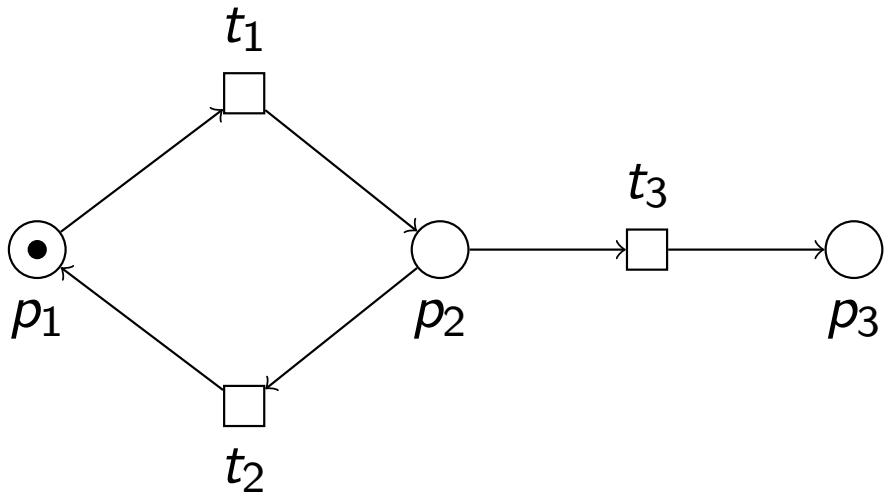
A *siphon* of a Petri net $(P, T, W)$ is a set of places $S$ such that:

$$\forall t \in T, S \cap succ(t) \neq \emptyset \Rightarrow S \cap pred(t) \neq \emptyset.$$

Once a siphon is unmarked, it remains unmarked.

$S = \{p_1, p_3\}$ is not a siphon because
$S \cap succ(t_3) = \{p_1, p_3\} \cap \{p_3\} = \{p_3\} \neq \emptyset$ but
$S \cap pred(t_3) = \{p_1, p_3\} \cap \{p_2\} = \emptyset$.

Here: $\emptyset$, $\{p_1, p_2\}$, $\{p_1, p_2, p_3\}$



# Petri net of a Boolean model

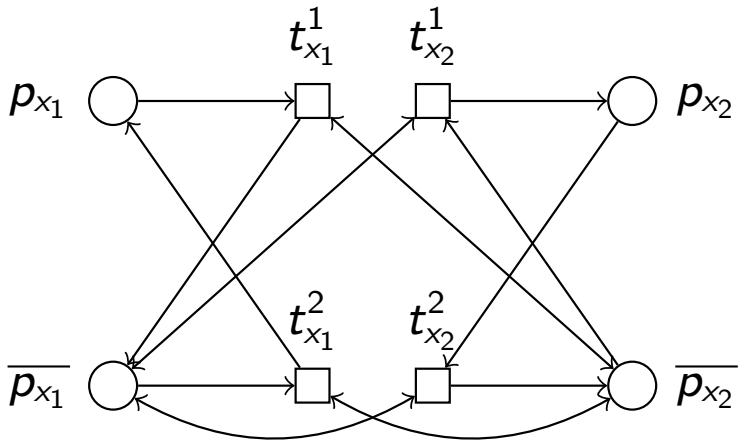
The original encoding was established in [Chaouiya et al., 2004].

Two places for each gene: $v \rightsquigarrow p_v, \overline{p_v}$

Solutions of $f_v \not\leftrightarrow v \rightsquigarrow$ transitions from $p_v$ to $\overline{p_v}$ (and back)

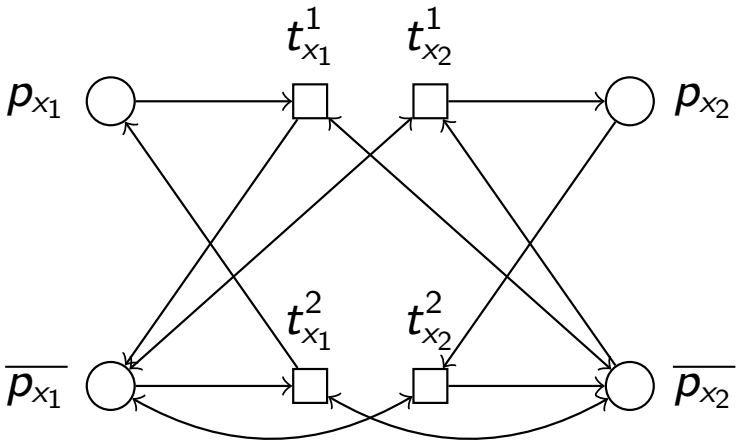
At any marking $m$ of the Petri net encoding a Boolean model, it always holds that $m(p_v) + m(\overline{p}_v) = 1$.

$$\begin{cases} f_1 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \\ f_2 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \end{cases}$$



# Conflict-free siphons

A siphon is called **conflict-free** if it does not contain both $p_v$ and $\overline{p_v}$ for all $v \in V$.




| Siphon | Conflict-free? |
|---|---|
| $\emptyset$ | yes |
| $\{\overline{p_{x_1}}, \overline{p_{x_2}}\}$ | yes |
| $\{p_{x_1}, \overline{p_{x_1}}\}$ | no |
| $\{p_{x_2}, \overline{p_{x_2}}\}$ | no |

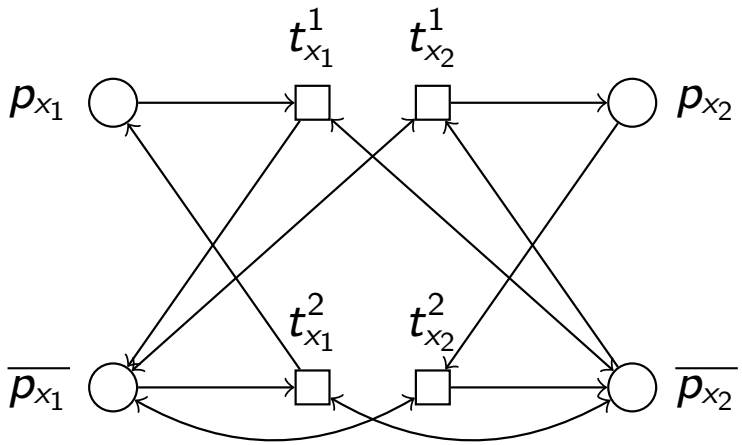
A conflict-free siphon is *maximal* if it is not a subset of any other conflict-free siphon.

# Conflict-free siphons are trap spaces

## Theorem 1

Let $\mathcal{M}$ be a Boolean model and $\mathcal{P}$ be its Petri net encoding. There is a one-to-one correspondence between the set of **trap spaces** of $\mathcal{M}$ and the set of **conflict-free siphons** of $\mathcal{P}$.

$$\begin{cases} f_1 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \\ f_2 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \end{cases}$$



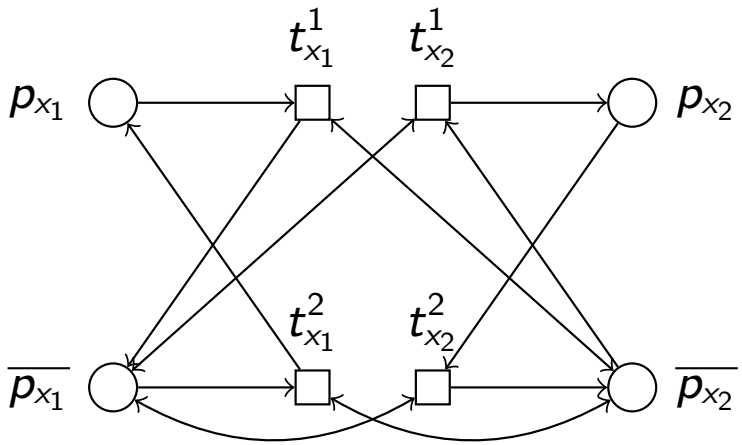

| Trap space | Conflict-free siphon |
|---|---|
| $\star\star$ | $\emptyset$ |
| 11 | $\{\overline{p_{x_1}}, \overline{p_{x_2}}\}$ |

# Maximal conflict-free siphons are minimal trap spaces

## Theorem 2

Let $\mathcal{M}$ be a Boolean model and $\mathcal{P}$ be its Petri net encoding. There is a one-to-one correspondence between the set of **minimal trap spaces** of $\mathcal{M}$ and the set of **maximal conflict-free siphons** of $\mathcal{P}$.

$$\begin{cases} f_1 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \\ f_2 = (x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2) \end{cases}$$




| Trap space | Conflict-free siphon |
|---|---|
| $\star\star$ | $\emptyset$ |
| 11 | $\{\overline{p_{x_1}}, \overline{p_{x_2}}\}$ |

# Proposed method for minimal trap space computation

From Theorem 2, we propose a new method for computing minimal trap spaces of a Boolean model $\mathcal{M}$.

- Build the Petri net encoding $\mathcal{P}$ of $\mathcal{M}$.
- Compute all maximal conflict-free siphons of $\mathcal{P}$.
- Convert the obtained maximal conflict-free siphons into the corresponding minimal trap spaces.

# Petri net transformation

Transforming a Boolean model into its Petri net encoding can be done via computing Disjunctive Normal Forms (DNF) of each Boolean function [Chatain et al., 2014].

Though this might appear quite computationally intensive it is important to remark first that contrary to the prime-implicants case, there is no need to find *minimal* DNFs.

We use the above transformation in our proposed method. The implementation uses BDDs[1].

[1] `https://github.com/cjdrake/pyeda`

# Maximal conflict-free siphon computation

Characterize all siphons of the encoded Petri net as a system of Boolean rules.

$$p \in S \Rightarrow \bigvee_{p' \in pred(t)} p' \in S, p \in P, t \in T, t \in pred(p)$$

Add to the system the Boolean rules representing the conflict-freeness.

$$p_v \in S \Rightarrow \overline{p_v} \notin S \wedge \overline{p_v} \in S \Rightarrow p_v \notin S, v \in V$$

Encode the system as an ASP.

Use an ASP solver (e.g., clingo [Gebser et al., 2011]) to compute all set-inclusion maximal answer sets of the ASP.

Set-maximality through "heuristics"

```
clingo --heuristic=Domain --enum-mod=domRec --dom-mod=3
```

# Locally-monotonic vs. non-locally-monotonic

A Boolean function is *locally-monotonic* if it can be represented by a formula in disjunctive normal form in which all occurrences of any given literal are either negated or non-negated [Anthony, 2001].

| Function | locally-monotonic? | non-locally-monotonic? |
|---|---|---|
| $x \wedge y$ | yes | no |
| $(x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2)$ | no | yes |

A Boolean model is said to be locally-monotonic if all its Boolean functions are locally-monotonic. Otherwise, this model is said to be non-locally-monotonic.

General models = locally-monotonic models + non-locally-monotonic models

# Locally-monotonic vs. non-locally-monotonic (cont.)

| Method | Applicable domain |
|---|---|
| mpbn [Paulevé et al., 2020] | locally-monotonic |
| PyBoolNet [Klarner et al., 2017] | general |
| our proposed method | general |

mpbn is specifically designed for exploiting the locally-monotonicity [Paulevé et al., 2020]. Hence, for locally-monotonic Boolean models, it has many advantages over other methods.

It is worth noting that the study [Noual et al., 2013] highlights the need for non-locally-monotonic Boolean models in both biological and theoretical aspects.

# PyBoolNet repo, 1000 first solutions

our proposed method

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| arellano_rootstem | 9 | 4 | 0.05 | 0.01 | 0.02 |
| calzone_cellfate | 28 | 27 | 0.03 | 0.02 | 0.03 |
| dahlhaus_neuroplastoma | 23 | 32 | 0.06 | 0.02 | 0.03 |
| | | ... | | | |
| jaoude_thdiff | 103 | >1000 | **1.92** | **1.32** | 0.20 |
| klamt_tcr | 40 | 8 | 0.04 | 0.01 | 0.04 |
| | | ... | | | |
| n5s3 | 4 | 3 | 0.03 | **NM** | 0.02 |
| | | ... | | | |
| randomnet_n15k3 | 15 | 3 | 0.04 | **NM** | 0.04 |
| randomnet_n7k3 | 7 | 10 | 0.03 | **NM** | 0.02 |
| remy_tumorigenesis | 34 | 25 | **2.05** | 0.04 | 0.06 |
| saadatpour_guardcell | 13 | 1 | 0.02 | 0.00 | 0.02 |
| selvaggio_emt | 56 | >1000 | **1.09** | **0.76** | 0.18 |
| tournier_apoptosis | 12 | 3 | 0.03 | 0.01 | 0.02 |
| xiao_wnt5a | 7 | 4 | 0.03 | 0.00 | 0.02 |
| zhang_tlgl | 60 | 156 | **0.22** | **0.22** | 0.09 |
| zhang_tlgl_v2 | 60 | 258 | 0.11 | **0.24** | 0.08 |

# PyBoolNet repo, 1000 first solutions

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| arellano_rootstem | 9 | 4 | 0.05 | 0.01 | 0.02 |
| calzone_cellfate | | 7 | 0.03 | 0.02 | 0.03 |
| dahlhaus_neuroplastoma | 23 | | | | 0.03 |
| | | ... | | | |
| jaoude_thdiff | 103 | >1000 | **1.92** | **1.32** | 0.20 |
| klamt_tcr | 40 | 8 | 0.04 | 0.01 | 0.04 |
| | | ... | | | |
| n5s3 | | | 0.03 | **NM** | 0.02 |
| | | ... | | | |
| randomnet_n15k3 | 15 | 3 | 0.04 | **NM** | 0.04 |
| randomnet_r | | | 0.03 | **NM** | 0.02 |
| remy_tumori | | | **2.05** | 0.04 | 0.06 |
| saadatpour_ | | | 0.02 | 0.00 | 0.02 |
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| zhang_tlgl | 60 | 156 | **0.22** | **0.22** | 0.09 |
| zhang_tlgl_v2 | 60 | 258 | 0.11 | **0.24** | 0.08 |

number of nodes

number of minimal trap spaces

non-monotonic model

significant difference to the best running time (in seconds)

# PyBoolNet repo, 1000 first solutions

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| arellano_rootstem | 9 | 4 | 0.05 | 0.01 | 0.02 |
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| | | ... | | | |
| jaoude_thdiff | 103 | >1000 | **1.92** | **1.32** | 0.20 |
| klamt_tcr | 40 | 8 | 0.04 | 0.01 | 0.04 |
| | | ... | | | |
| n | | | | | 2 |
| randomnet_n15k3 | 15 | 3 | 0.04 | **NM** | 0.04 |
| randomnet_n7k3 | 7 | 10 | 0.03 | **NM** | 0.02 |
| remy_tumorigenesis | 34 | 25 | **2.05** | 0.04 | 0.06 |
| saadatpour_guardcell | 13 | 1 | 0.02 | 0.00 | 0.02 |
| selvaggio_emt | 56 | >1000 | **1.09** | **0.76** | 0.18 |
| tournier_apoptosis | 12 | 3 | 0.03 | 0.01 | 0.02 |
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| zhang_tlgl_v2 | 60 | 258 | 0.11 | **0.24** | 0.08 |

All three methods are comparable with all minimal trap spaces found very fast because the models are quite small.

# PyBoolNet repo, 1000 first solutions

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| arellano_rootstem | 9 | 4 | 0.05 | 0.01 | 0.02 |
| calzone_cellfate | 28 | 27 | 0.03 | 0.02 | 0.03 |
| dahlhaus_neuroplastoma | 23 | 32 | 0.06 | 0.02 | 0.03 |
| | | ... | | | |
| jaoude_tbdiff | 103 | >1000 | **1.02** | **1.32** | 0.20 |
| k | | | | | 4 |
| n5s3 | 4 | 3 | 0.03 | **NM** | 0.02 |
| | | ... | | | |
| randomnet_n15k3 | 15 | 3 | 0.04 | **NM** | 0.04 |
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| zhang_tlgl | 60 | 156 | **0.22** | **0.22** | 0.09 |
| zhang_tlgl_v2 | 60 | 258 | 0.11 | **0.24** | 0.08 |

For 3 of the 29 models, mpbn did not give any answer because these models are non-locally-monotonic.

# PyBoolNet repo, 1000 first solutions

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| arellano_rootstem | 9 | 4 | 0.05 | 0.01 | 0.02 |
| calzone_cellfate | 28 | 27 | 0.03 | 0.02 | 0.03 |
| dahlhaus_neuroplastoma | 23 | 32 | 0.06 | 0.02 | 0.03 |
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| jaoude_thdiff | 103 | >1000 | **1.92** | **1.32** | 0.20 |
| klamt_tcr | 40 | 8 | 0.04 | 0.01 | 0.04 |
| n | | | | | 2 |
| | | ... | | | |
| randomnet_n15k3 | 15 | 3 | 0.04 | **NM** | 0.04 |
| randomnet_n7k3 | 7 | 10 | 0.03 | **NM** | 0.02 |
| remy_tumorigenesis | 34 | 25 | **2.05** | 0.04 | 0.06 |
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| zhang_tlgl | 60 | 156 | **0.22** | **0.22** | 0.09 |
| zhang_tlgl_v2 | 60 | 258 | 0.11 | **0.24** | 0.08 |

On every model that was a bit challenging for PyBoolNet or mpbn, the new method is more efficient with significant speedups.

# Large models from literature, 2 min timeout

| model | $n$ | $\|M\|$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| inflammatory-bowel | 47 | 1 | **DNF** | **NM** | 1.71 |
| T-LGL-survival | 61 | 318 | 0.23 | 0.30 | 0.10 |
| butanol-production | 66 | >1000 | 0.20 | **0.88** | 0.12 |
| colon-cancer | 70 | 10 | **0.07** | **0.04** | 0.06 |
| mast-cell-activation | 73 | >1000 | **0.16** | **0.89** | 0.13 |
| IL-6-signalling | 86 | >1000 | 0.17 | **1.01** | 0.13 |
| Corral-ThIL-17-diff | 92 | >1000 | **DNF** | **1.18** | 0.18 |
| Korkut-2015 | 99 | >1000 | **DNF** | 1.36 | 0.39 |
| adhesion-cip-migration | 121 | 78 | **28.77** | 0.34 | 0.25 |
| interferon-1 | 121 | >1000 | **2.82** | **1.42** | 0.17 |
| TCR-TLR5-signaling | 130 | 48 | **0.54** | 0.18 | 0.13 |
| influenza-replication | 131 | >1000 | **8.75** | **1.54** | 0.20 |
| prostate-cancer | 133 | >1000 | **DNF** | **2.71** | 0.38 |
| HIV-1 | 138 | >1000 | **DNF** | **11.77** | 0.36 |
| fibroblasts | 139 | >1000 | **DNF** | **NM** | 0.42 |
| HMOX-1-pathway | 145 | >1000 | **1.65** | **1.98** | 0.20 |

# Large models from literature, 2 min timeout (cont.)

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| | | ... | | | |
| MAPK | 181 | >1000 | **86.81** | **2.64** | 0.30 |
| er-stress | 182 | >1000 | **11.13** | **2.26** | 0.24 |
| cascade-3 | 183 | 1 | **DNF** | 0.33 | 0.24 |
| CHO-2016 | 200 | >1000 | **DNF** | **3.36** | 0.36 |
| T-cell-check-point | 218 | >1000 | **28.83** | **NM** | 0.38 |
| ErbB-receptor-signaling | 247 | >1000 | **DNF** | **NM** | 1.06 |
| macrophage-activation | 321 | >1000 | **10.47** | **3.86** | 0.50 |
| cholocystokinin | 383 | >1000 | **2.83** | **4.46** | 0.49 |
| Alzheimer | 762 | >1000 | **DNF** | **NM** | 0.99 |
| KEGG-network | 1659 | >1000 | **DNF** | 21.57 | 30.22 |
| human-network | 1953 | >1000 | **DNF** | 25.19 | 21.91 |
| SN-5 | 2746 | >1000 | **DNF** | 37.54 | 45.57 |
| turei-2016 | 4691 | >1000 | **DNF** | 119.98 | **DNF** |

# Large models from literature, 2 min timeout (cont.)

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| | | ... | | | |
| MAPK | 181 | >1000 | **86.81** | **2.64** | 0.30 |
| er-stress | 182 | >1000 | **11.13** | **2.26** | 0.24 |
| cascade-3 | 183 | 1 | **DNF** | 0.33 | 0.24 |
| CHO-2016 | 200 | >1000 | **DNF** | **3.36** | 0.36 |
| T-cell-check-point | 218 | >1000 | **28.83** | **NM** | 0.38 |
| These models are quite big (in size), complex (i.e., having high average in-degree) and most of them have never been fully analyzed. | | | | | |
| cholocystokinin | 383 | >1000 | **2.83** | **4.46** | 0.49 |
| Alzheimer | 762 | >1000 | **DNF** | **NM** | 0.99 |
| KEGG-network | 1659 | >1000 | **DNF** | 21.57 | 30.22 |
| human-network | 1953 | >1000 | **DNF** | 25.19 | 21.91 |
| SN-5 | 2746 | >1000 | **DNF** | 37.54 | 45.57 |
| turei-2016 | 4691 | >1000 | **DNF** | 119.98 | **DNF** |

# Large models from literature, 2 min timeout (cont.)

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| | | ... | | | |
| MAPK | 181 | >1000 | **86.81** | **2.64** | 0.30 |
| er-stress | 182 | >1000 | **11.13** | **2.26** | 0.24 |
| cascade-3 | 183 | 1 | **DNF** | 0.33 | 0.24 |
| CHO-2016 | 200 | >1000 | **DNF** | **3.36** | 0.36 |
| T-cell-check-point | 218 | >1000 | **28.83** | **NM** | 0.38 |
| For 6 of the 33 models, mpbn did not give any answer because these models are non-locally-monotonic. | | | | | |
| cholocystokinin | 383 | >1000 | **2.83** | **4.46** | 0.49 |
| Alzheimer | 762 | >1000 | **DNF** | **NM** | 0.99 |
| KEGG-network | 1659 | >1000 | **DNF** | 21.57 | 30.22 |
| human-network | 1953 | >1000 | **DNF** | 25.19 | 21.91 |
| SN-5 | 2746 | >1000 | **DNF** | 37.54 | 45.57 |
| turei-2016 | 4691 | >1000 | **DNF** | 119.98 | **DNF** |

# Large models from literature, 2 min timeout (cont.)

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| | | ... | | | |
| MAPK | 181 | >1000 | **86.81** | **2.64** | 0.30 |
| er-stress | 182 | >1000 | **11.13** | **2.26** | 0.24 |
| cascade-3 | 183 | 1 | **DNF** | 0.33 | 0.24 |
| CHO-2016 | 200 | >1000 | **DNF** | **3.36** | 0.36 |
| T-cell-check-point | 218 | >1000 | **28.83** | **NM** | 0.38 |
| Alzheimer | 762 | >1000 | **DNF** | **NM** | 0.99 |
| KEGG-network | 1659 | >1000 | **DNF** | 21.57 | 30.22 |
| human-network | 1953 | >1000 | **DNF** | 25.19 | 21.91 |
| SN-5 | 2746 | >1000 | **DNF** | 37.54 | 45.57 |
| turei-2016 | 4691 | >1000 | **DNF** | 119.98 | **DNF** |

For 26 of the 33 models where both mpbn and Trappist returned the answers, they are comparable in computation time, though surprisingly mpbn appears a bit slower on average.

# Large models from literature, 2 min timeout (cont.)

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| | | ... | | | |
| MAPK | 181 | >1000 | **86.81** | **2.64** | 0.30 |
| er-stress | 182 | >1000 | **11.13** | **2.26** | 0.24 |
| cascade-3 | 183 | 1 | **DNF** | 0.33 | 0.24 |
| CHO-2016 | 200 | >1000 | **DNF** | **3.36** | 0.36 |
| T-cell-check-point | 218 | >1000 | **28.83** | **NM** | 0.38 |
| Alzheimer | 762 | >1000 | **DNF** | **NM** | 0.99 |
| KEGG-network | 1659 | >1000 | **DNF** | 21.57 | 30.22 |
| human-network | 1953 | >1000 | **DNF** | 25.19 | 21.91 |
| SN-5 | 2746 | >1000 | **DNF** | 37.54 | 45.57 |
| turei-2016 | 4691 | >1000 | **DNF** | 119.98 | **DNF** |

Note however that mpbn was the only tool to provide a solution for the turei model within two minutes, thus confirming its advantages for locally-monotonic models.

# Large models from literature, 2 min timeout (cont.)

| model | $n$ | $\lvert M\rvert$ | PyBoolNet | mpbn | Trappist |
|---|---|---|---|---|---|
| | | ... | | | |
| MAPK | 181 | >1000 | **86.81** | **2.64** | 0.30 |
| er-stress | 182 | >1000 | **11.13** | **2.26** | 0.24 |
| cascade-3 | 183 | 1 | **DNF** | 0.33 | 0.24 |
| CHO-2016 | 200 | >1000 | **DNF** | **3.36** | 0.36 |
| T-cell-check-point | 218 | >1000 | **28.83** | **NM** | 0.38 |
| Alzheimer | 762 | >1000 | **DNF** | **NM** | 0.99 |
| KEGG-network | 1659 | >1000 | **DNF** | 21.57 | 30.22 |
| human-network | 1953 | >1000 | **DNF** | 25.19 | 21.91 |
| SN-5 | 2746 | >1000 | **DNF** | 37.54 | 45.57 |
| turei-2016 | 4691 | >1000 | **DNF** | 119.98 | **DNF** |

The proposed method vastly outperforms PyBoolNet in computational time, on each and every model, and sometimes with orders of magnitude of difference.

# Conclusion

Minimal trap spaces are important in Boolean model analysis.

We linked the concept of trap spaces in the Boolean networks field and the concept of siphons on the Petri nets field.

We proposed a new method for the computation of minimal trap spaces of Boolean models.

The evaluation on large models from the literature shows that

- our method can scale up much better than the state-of-the-art prime-implicants based techniques for non-locally-monotonic models;
- our method is comparable to mpbn for locally-monotonic models.

We believe that this opens up the way to a much better analysis of large Boolean models, hence **biological systems**.

# Future work

Improve our method to deal with larger and more complex models.

Extend our method to that for multi-level logical models, which can model biological systems better.

Finally, we think that the links between Petri nets and Boolean models that we stumbled upon in this method might have deeper roots. Exploring those connections might lead both to interesting topics of research for Petri nets, like a notion of trap spaces, and for Boolean models.

Thank you for your attention!

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